HYPER VERSION

Received: *January 5, 2001,* Accepted: *February 19, 2001*

A D2-brane realization of Maxwell-Chern-Simons-Higgs systems

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Abstract: We show that $\mathcal{N} = 2$ supersymmetric Maxwell-Chern-Simons-Higgs systems in three dimension can be realized as gauge theories on a D2-brane in D8-branes background with a non-zero B-field. We reproduce a potential of Coulomb branch of the Chern-Simons theory as a potential of a D2-brane in a classical D8-brane solution and show that each Coulomb vacuum is realized by a D2-brane stabilized in the bulk at a certain distance from D8-branes.

Keywords: Chern-Simons Theories, Superstrings and Heterotic Strings, D-branes.

JHEP02(2001)035

Contents

1. Introduction

Chern-Simons theories in three-dimension [1] have many interesting properties: rich phase structures, existence of solitons, etc. In this paper, we discuss their brane realization.

One way to realize three-dimensional gauge theories in string theory is to use Hanany-Witten type configurations. $\mathcal{N} = 4$ gauge theories are realized on D3-branes stretched between parallel NS5-branes [2]. We can decrease the number of supersymmetries and can introduce a non-zero Chern-Simons coupling by replacing NS5-branes by (p, q) 5-branes and putting them at appropriate angles [3]–[8]. Using these configurations, moduli space, phase structures and solitons of the theories are studied in geometrical ways.

We can also realize three-dimensional gauge theories as theories on D2-branes [9, 10]. For example, an $\mathcal{N} = 4$ $U(1)$ gauge theory with N_f flavors is realized on a D2-brane in N_f D6-branes background. However, unlike the Hanany-Witten type configurations, we come up against a problem if we attempt to introduce a Chern-Simons term. The D2-brane action involves the following term [11]

$$S = \frac{\Lambda}{4\pi} \int A \wedge dA \,, \tag{1.1}$$

where Λ is a cosmological constant quantized in integer units. Because we need to use this coupling to introduce a nonzero Chern-Simons coupling, we should put D2-branes in a massive IIA or, equivalently, a D8-brane background. If we put a D2-brane parallel to a D8-brane, however, the configuration becomes non-BPS and unstable. In order to keep supersymmetries unbroken, we have to take one direction on the D2-brane to be a dilatonic direction perpendicular to the D8-brane. Although $\mathcal{N} = (8, 0)$ supersymmetry is realized in this case, the three-dimensional Poincaré invariance is broken [12].

Recently, Mihailescu, Park and Tran [13] and Witten [14] showed that a Dp-D$(p+2)$ system, which is ordinary non-BPS, can be made a BPS configuration by introducing an appropriate B-field. In this paper, inspired by their idea, we study a relation between a D2-D6 configuration with a non-zero B-field and a Maxwell-Chern-Simons-Higgs system expected to be realized on the D2-brane.

2. Maxwell-Chern-Simons-Higgs systems

We consider a D2-D8 system with a non-zero B-field. Let us assume that N_f D8-branes are located at $x^9 = q_I$ $(I = 1, 2, \ldots, N_f)$ and one D2-brane spreads along the x^0, x^1 and x^2 directions. We turn on the following components of the B-field.

$$\frac{B_{36}}{T} = \tan\theta_1\,, \quad \frac{B_{47}}{T} = \tan\theta_2\,, \quad \frac{B_{58}}{T} = \tan\theta_3\,, \qquad (0 \le \theta_i \le \pi)\,, \tag{2.1}$$

where $T = 1/(2\pi l_s^2)$ is the string tension. This configuration is T-dual to a D3-D9 system in [13] and a D0-D6 system in [14]. Therefore, we can divert results there to analysis of our D2-D8 system. A BPS condition for this system is

$$e^{i(\theta_1+\theta_2+\theta_3)} = -i\,. \tag{2.2}$$

We define a parameter r by

$$r = \theta_1 + \theta_2 + \theta_3 - \frac{3\pi}{2}\,. \tag{2.3}$$

This parameter corresponds to the r in [14]. We have fixed some ambiguities for definitions of parameters differently from [14] for later convenience. Near the BPS point $r = 0$, an $\mathcal{N} = 2$ U(1) gauge theory is expected to be realized on the D2-brane. The field content of this theory is one U(1) vector multiplet (a, A_μ, λ, D) from 2-2 strings and N_f chiral multiplets (ϕ_I, ψ_I, F_I) $(I = 1, 2, \ldots, N_f)$ from 2-8 strings. Precisely, there are three more chiral multiplets from 2-2 strings representing fluctuations of the D2-brane parallel to the D8-branes. Because they are neutral under the U(1) and decoupled from other fields, we shall neglect them in what follows.

Let us consider an $\mathcal{N} = 2$ supersymmetric action for these fields. Kinetic terms of vector and chiral multiplets are

$$\mathcal{L}_{\text{vector-kin}} = \frac{1}{g^2}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{1}{2}D^2\right), \tag{2.4}$$

$$\mathcal{L}_{\text{chiral-kin}} = \frac{1}{g^2}\sum_{I=1}^{N_f}\left(-\mathcal{D}_\mu\overline{\phi}_I\mathcal{D}^\mu\phi_I + \overline{F}_I F_I + D|\phi_I|^2 + a\overline{F}_I\phi_I + a\overline{\phi}_I F_I\right). \tag{2.5}$$

(We are now interested in only bosonic fields.) For chiral multiplets, we can add mass terms

$$\mathcal{L}_{\text{chiral}-\text{mass}} = -\frac{1}{g^2}\sum_{I=1}^{N_f} M_I(\phi_I \overline{F}_I + \overline{\phi}_I F_I). \tag{2.6}$$

Because gauge group is U(1), we can introduce a Fayet-Iliopoulous term

$$\mathcal{L}_{\text{FI}} = \frac{\xi}{g^2} D. \tag{2.7}$$

Furthermore, in three-dimension, a Chern-Simons term is allowed.

$$\mathcal{L}_{\text{CS}} = \frac{k}{4\pi}\epsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma\,, \tag{2.8}$$

where k is an integer-valued quantized Chern-Simons coupling. The equation of motion for the gauge field is

$$\partial_\mu F^{\mu\lambda} + \frac{\mu}{2}\epsilon^{\lambda\mu\nu}F_{\mu\nu} = \text{current}\,, \qquad \left(\mu = \frac{kg^2}{2\pi}\right). \tag{2.9}$$

Because the second term on the left hand side plays a role of a 'mass term' of the gauge field, we should introduce mass terms also for other component fields in the vector multiplet in order to make the action supersymmetric [15, 16]

$$\mathcal{L}_{\text{vector}-\text{mass}} = -\frac{\mu}{g^2} aD \tag{2.10}$$

Gathering all and eliminating auxiliary fields F_I and D, we obtain

$$V(a,\phi_I) = \frac{1}{g^2}\left[\frac{1}{2}\left(\sum_{I=1}^{N_f}|\phi_I|^2 - \mu a + \xi\right)^2 + \sum_{I=1}^{N_f}(a - M_I)^2|\phi_I|^2\right]. \tag{2.11}$$

This is a classical potential. Due to a quantum effect, the Chern-Simons coupling k get a correction. Because k is quantized in integer units, it cannot vary continuously. By calculating a one loop diagram of fermion fields ψ_I, we find the coupling $k(a)$ jumps by one at $a = M_I$ for each I [17]. Therefore, $\xi - \mu a$ in the classical potential (2.11) should be replaced by a function $h(a)$ satisfying

$$\frac{d}{da}h(a) = -\frac{g^2}{2\pi}k(a)\,. \tag{2.12}$$

Finally, we obtain a quantum potential for this theory.

$$V(a,\phi_I) = \frac{1}{g^2}\left[\frac{1}{2}\left(\sum_{I=1}^{N_f}|\phi_I|^2 + h(a)\right)^2 + \sum_{I=1}^{N_f}(a - M_I)^2|\phi_I|^2\right]. \tag{2.13}$$

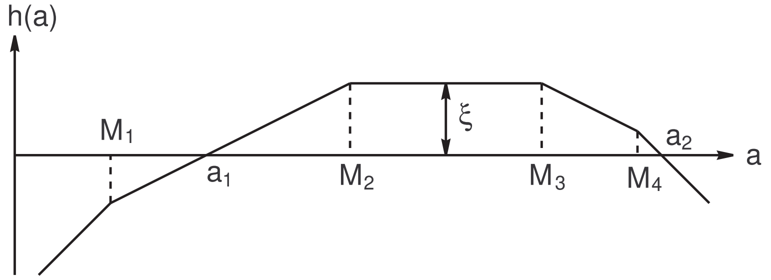

Figure 1: An example of the function $h(a)$. We define ξ as a height of a 'plateau' of the function $h(a)$. In this example, there are one Higgs vacuum ($a = M_1, |\phi_1| = \sqrt{-h(M_1)}$) and two Coulomb vacua ($a = a_1, \phi_I = 0$) and ($a = a_2, \phi_I = 0$).

In what follows, we shall use a parameter ξ as what represents a height of a 'plateau' of the function $h(a)$ (figure 1). (It is not necessary for the function $h(a)$ to have a plateau. It can be monotonically increasing or monotonically decreasing. We assume its existence just for convenience in explanations.)

Next, let us consider vacua of this theory. There are two kinds of vacua: 'Higgs vacua' and 'Coulomb vacua'.

Higgs vacua. An expansion of the potential (2.13) in ϕ_I is

$$V(a, \phi_I) = \frac{h^2(a)}{2g^2} + \frac{1}{g^2} \sum_{I=1}^{N_f} (h(a) + (a - M_I)^2)|\phi_I|^2 + \mathcal{O}(\phi_I^4)\,. \tag{2.14}$$

Roughly speaking, ϕ_I is tachyonic around $a = M_I$ if $h(M_I)$ is negative. At $a = M_I$, a potential for ϕ_I is $V = (1/2g^2)(|\phi_I|^2 + h(a))^2$ and there is supersymmetric vacuum $|\phi_I| = \sqrt{-h(M_I)}$ if $h(M_I) < 0$. This vacuum breaks the gauge symmetry. As is mentioned in [14] for D0-D6 systems, Higgs vacuum with $\phi_I \neq 0$ is regarded as a true bound state of the D2-brane and the I-th D8-brane. Although it would be interesting problem how to realize this bound state as a classical supergravity solution, we will not argue this in this paper.

Coulomb vacua. In the Coulomb branch $\phi_I = 0$, the potential is

$$V(a) = \frac{h^2(a)}{2g^2}\,. \tag{2.15}$$

There are three phases. If $\xi < 0$, the potential $V(a)$ is everywhere positive. In this case there are supersymmetric Higgs vacua as we mentioned above and a state on the Coulomb branch decays into one of the Higgs vacua. If $\xi = 0$, we have a continuous set of vacua on the plateau of the function $h(a)$. All these vacua are supersymmetric and have an unbroken gauge symmetry. For $\xi > 0$, we have supersymmetric Coulomb vacua at a satisfying $h(a) = 0$. On these vacua, both the supersymmetry and the

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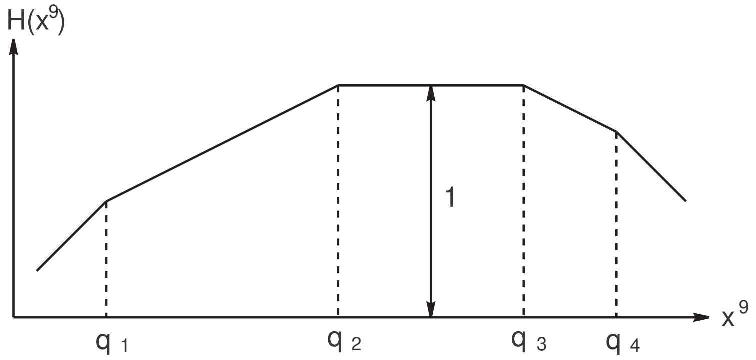

Figure 2: The harmonic function $H(x^9)$. Each D8-brane is located at $x^9 = q_I$.

gauge symmetry are unbroken. The fact that only specific values of scalar field a are chosen suggests an interesting phenomenon in string theory. It implies that the D2-brane is stabilized in the bulk at a certain distance from the D8-branes. In the next section, we show that the potential $V(a)$ is reproduced as one for a D2-brane and the stabilization actually takes place.

3. A classical D8-brane solution with B-field

In this section, we construct a classical solution for D8-branes in $B \neq 0$ and reproduce the potential (2.15) as a potential for a D2-brane in the D8-brane background.

We can make a D8-brane solution with a non-zero B-field by T-dualizing a smeared D5-brane solution. Let us begin with the following D5-brane solution.

$$ds^2 = H^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H^{1/2}\delta_{ij}dx^i dx^j\,, \tag{3.1}$$

$$e^\Phi = g_{\rm str}H^{-1/2}\,, \tag{3.2}$$

$$C_{012345} = \frac{2\pi}{g_{\rm str}}(2\pi l_s)^{-6}H^{-1}\,, \tag{3.3}$$

where $\mu,\nu = 0,1,\ldots,5$ and $i,j = 6,7,8,9$. The harmonic function H satisfies the following Laplace equation in the transverse directions.

$$\Delta_4 H = -(2\pi l_s)^2 g_{\rm str}\rho^{(4)}\,, \tag{3.4}$$

where $\rho^{(4)}$ is a D5-brane density which is now taken to be

$$\rho^{(4)} = \frac{1}{(2\pi l_s)^3(-\prod_{a=1}^3\cos\theta_a)}\sum_I \delta(x^9 - q_I)\,. \tag{3.5}$$

The $1/\prod\cos\theta_a$ factor is necessary to compensate a change of the brane density due to the rotation which we will do next. (We assume that one of θ_a is larger than

$\pi/2$ and other two are smaller than $\pi/2$. Therefore this density is positive.) By integrating (3.4) once, we obtain

$$\frac{d}{dx^9}H(x^9) = -\frac{g_{\rm str}}{2\pi l_s(-\prod_{a=1}^3 \cos\theta_a)}\Lambda(x^9)\,, \tag{3.6}$$

where $\Lambda(x^9)$ is a function representing the quantized cosmological constant after the T-duality transformation. Because $\Lambda(x^9)$ jumps by one as x^9 crosses the position of each D8-brane and is identified with the Chern-Simons coupling $k(a)$, the function $H(x^9)$ has a similar form to $h(a)$ (figure 2). These two functions, however, are not proportional to each other. $H(x^9)$ must not be negative while $h(a)$ may. Let us define $g_{\rm str}$ as an expectation value of e^Φ on the plateau of the function $H(x^9)$. Then, the constant part of $H(x^9)$ is fixed by (3.2), such that $H(x^9) = 1$ on the plateau.

Rotating this solution on 3-6, 4-7, and 5-8 planes by angles θ_1, θ_2 and θ_3, respectively, we obtain the following metric and the R-R 6-form potential.

$$\begin{aligned} ds^2 &= H^{-1/2}(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2) + H^{1/2}(dx^9)^2 + \\ &\quad + \sum_{a=1}^3 (H^{-1/2}\cos^2\theta_a + H^{1/2}\sin^2\theta_a)(dx^{a+2})^2 + \\ &\quad + \sum_{a=1}^3 (H^{-1/2}\sin^2\theta_a + H^{1/2}\cos^2\theta_a)(dx^{a+5})^2 + \\ &\quad +2\sum_{a=1}^3 \cos\theta_a \sin\theta_a (H^{-1/2} - H^{1/2}) dx^{a+2}dx^{a+5}\,, \qquad (3.7) \\ C_{012678} &= \frac{2\pi}{g_{\rm str}}(2\pi l_s)^{-6} H^{-1}\sin\theta_1 \sin\theta_2 \sin\theta_3\,. \qquad (3.8) \\ C_{012345} &= \frac{2\pi}{g_{\rm str}}(2\pi l_s)^{-6} H^{-1}\cos\theta_1 \cos\theta_2 \cos\theta_3\,. \qquad (3.9) \end{aligned}$$

Other components of the R-R 6-form potential are irrelevant to our arguments.

Compactifying the x^6, x^7 and x^8 directions on a rectangular $\mathbf{T}^3$ with all period $2\pi l_s$ and carrying out T-duality transformation by relations

$$\widetilde{G}_{a+2,a+2} = G_{a+2,a+2} - \frac{G_{a+2,a+5}^2}{G_{a+5,a+5}}\,, \quad \widetilde{G}_{a+5,a+5} = \frac{1}{G_{a+5,a+5}}\,, \quad (a=1,2,3)\,, \tag{3.10}$$

we obtain the following metric for a D8-brane solution with a nonzero B-field.

$$\widetilde{G}_{\mu\nu} = H^{-1/2}\eta_{\mu\nu} \quad (\mu,\nu = 0,1,2)\,, \quad \widetilde{G}_{a+2,a+2} = \widetilde{G}_{a+5,a+5} = \frac{H^{1/2}}{F_a}\,, \quad \widetilde{G}_{99} = H^{1/2}\,, \tag{3.11}$$

where the function F_a is defined by

$$F_a = \sin^2\theta_a + H\cos^2\theta_a\,. \tag{3.12}$$

The dual dilaton field is

$$e^{\widetilde{\Phi}} = e^{\Phi} G_{66}^{-1/2} G_{77}^{-1/2} G_{88}^{-1/2} = g_{\rm str} H^{1/4} F_1^{-1/2} F_2^{-1/2} F_3^{-1/2} \,. \tag{3.13}$$

From (3.8), we obtain a component of the R-R 3-form potential coupling to the D2-brane as

$$\widetilde{C}_{012} = T_{D2} H^{-1} \sin\theta_1 \sin\theta_2 \sin\theta_3 \,. \tag{3.14}$$

where $T_{D2} = 1/\{(2\pi)^2 l_s^3 g_{\rm str}\}$ is the D2-brane tension on the plateau $H(x^9) = 1$. The zero-form R-R field strength $\widetilde{G}$ is T-dual to G_{678}, which is the field strength of R-R two-form field dual to (3.9).

$$\widetilde{G} = (2\pi l_s)^3 G_{678} = \frac{2\pi}{g_{\rm str}} (2\pi l_s) H' \cos\theta_1 \cos\theta_2 \cos\theta_3 = 2\pi\Lambda \,. \tag{3.15}$$

Λ is certainly identified with the cosmological constant $\widetilde{G}/2\pi$ quantized in integer units as we mentioned above. Using the solution we have obtained, a potential for a D2-brane in this background is

$$\begin{aligned} V &= \frac{1}{(2\pi)^2 l_s^3 e^{\widetilde{\Phi}}} \sqrt{-\widetilde{G}_{00} \widetilde{G}_{11} \widetilde{G}_{22}} - \widetilde{C}_{012} \\ &= T_{D2} \frac{1}{H} \left[\prod_{a=1}^{3} (\sin^2\theta_a + H \cos^2\theta_a)^{1/2} - \prod_{a=1}^{3} \sin\theta_a \right] \end{aligned} \tag{3.16}$$

The following relations hold between parameters in the gauge theory and ones in string theory.

$$k = \Lambda, \qquad a = T x^9, \qquad \frac{1}{g^2} = \frac{T_{D2}}{T^2} \,. \tag{3.17}$$

Comparing (2.12) and (3.6), we obtain the following relation between $h(a)$ and $H(x^9)$.

$$h(a) - \xi = \left(- \prod_{a=1}^{3} \cos\theta_a \right) T (H(a/T) - 1) \,. \tag{3.18}$$

In order to take the field theory limit, let us expand the potential (3.16) around $H = 1$.

$$\frac{V}{T_{D2}} = A(\theta_a) + B(\theta_a)\epsilon + C(\theta_a)\epsilon^2 + \mathcal{O}\epsilon^3 \,. \tag{3.19}$$

where $\epsilon = 1 - H$. Near the BPS point $r = 0$, the coefficients $B(\theta_a)$ and $C(\theta_a)$ are expanded in r as

$$B(\theta_a) = r \cos\theta_1 \cos\theta_2 \cos\theta_3 + \mathcal{O}(r^2) \,, \qquad C(\theta_a) = \frac{1}{2} \cos^2\theta_1 \cos^2\theta_2 \cos^2\theta_3 + \mathcal{O}(r) \,. \tag{3.20}$$

Therefore, the potential is rewritten as

$$V = \frac{T_{D2}}{2T^2}(Tr + T\epsilon \cos\theta_1 \cos\theta_2 \cos\theta_3)^2 \,, \tag{3.21}$$

up to a constant. Using (3.17) and (3.18), we can rewrite this potential in terms of variables in the field theory except the parameter r.

$$V = \frac{1}{2g^2}(Tr + \xi - h(a))^2 \,, \tag{3.22}$$

This coincides with the potential (2.15) if r relates to the FI-parameter ξ by

$$\xi = -Tr \,. \tag{3.23}$$

Now we have shown that the potential in the Coulomb branch of Maxwell-Chern-Simons-Higgs system is reproduced as a potential for a D2-brane on the D8-brane background. When $\xi > 0$, which corresponds to $r < 0$, we have Coulomb vacua at points with $h(a) = 0$. Each of these vacua is realized by a D2-brane stabilized in the bulk at a minimum of the potential. Note that such a stabilization is impossible in case with a vanishing B-field. This can be seen by putting θ_a to be zero in (3.16). In this case, $V \propto H^{1/2}$ and there is no stable point.

4. Discussions

In this paper, we showed that Maxwell-Chern-Simons-Higgs systems are realized as gauge theories on a D2-brane in a D8-brane background with a nonzero B-field. The potential of Coulomb branch is reproduced as a potential for a D2-brane.

There are some open questions. We argued only the Coulomb branch in a supergravity framework. How can we treat the Higgs branch? One way to do it is to use a noncommutative gauge theory on D8-branes. In the decoupling limit, $r = \sum_a (\theta_a - \pi/2)$ is proportional to T^{-1}. Let us take a limit in which each $\theta_a - \pi/2$ is proportional to T^{-1}. From (2.1), we obtain $B_{36}, B_{47}, B_{58} \propto T^2$ and this is what we take in order to realize a noncommutative gauge theory on the D8-branes [18]. Higgs vacua are expected to be realized as noncommutative solitons on the D8-branes. By noncommutative parameters θ_{ij}, the FI parameter ξ is represented as

$$-\xi = \frac{T^2}{B_{36}} + \frac{T^2}{B_{47}} + \frac{T^2}{B_{58}} = \frac{1}{\theta_{36}} + \frac{1}{\theta_{47}} + \frac{1}{\theta_{58}} \,. \tag{4.1}$$

Although Higgs vacua are always supersymmetric, noncommutative soliton solutions are BPS only when $\xi = 0$ [13, 14]. These do not contradict to each other because in the limit $\theta_a - \pi/2 \propto T^{-1}$ the expansions (3.20) are valid only when $\xi = r = 0$. For non-zero ξ, it is not clear whether we can use solitonic solutions on the D8-branes for the purpose of analysis of gauge theories on the D2-branes.

A problem on supersymmetry exists for the Coulomb branch, too. For $\xi > 0$, there are supersymmetric Coulomb vacua which are realized by stabilized D2-branes. The D2-D8 systems, however, do not have any explicit unbroken supersymmetry.

Acknowledgments

The author would like to thank K. Ohta. Discussions with him at Summer Institute 2000 at Yamanashi, Japan was very helpful for this work.

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